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## ВПЛИВ ПАРАЗИТНОЇ ІНДУКТИВНОСТІ НАКОПИЧУВАЛЬНОГО КОНДЕНСАТОРА ТА ЄМНОСТІ НАВАНТАЖЕННЯ НА ПЕРЕХІДНІ ПРОЦЕСИ У ВИСОКОВОЛЬТНИХ УСТАНОВКАХ ІЗ ІМПУЛЬСНИМИ ТРАНСФОРМАТОРАМИ

У статті розглядається вплив паразитних індуктивностей накопичувального конденсатора і ємності об'єкта контролю на струми в первинній та вторинній обмотках імпульсного трансформатора. Крім того розглядається вплив паразитних індуктивностей на напругу на навантажувальній ємності об'єкта контролю, а також на ємності накопичувального конденсатора. Згідно з результатами проведеного моделювання паразитна індуктивність об'єкта контролю має більш значний вплив на форму напруги на ємності навантаження ніж паразитна індуктивність накопичувального конденсатора.

**Ключові слова:** характеристичний поліном, взаємна індуктивність, перетворення Лапласа.

***И. А. КОСТЮКОВ***

## ВЛИЯНИЕ ПАРАЗИТНОЙ ИНДУКТИВНОСТИ НАКОПИТЕЛЬНОГО КОНДЕНСАТОРА И НАГРУЗОЧНОЙ ЕМКОСТИ НА ПЕРЕХОДНЫЕ ПРОЦЕССЫ В ВЫСОКОВОЛЬТНЫХ УСТАНОВКАХ С ИМПУЛЬСНЫМИ ТРАНСФОРМАТОРАМИ

В статье рассматривается влияние паразитных индуктивностей накопительного конденсатора и емкости тестируемого объекта контроля на токи в первичной и вторичной обмотке импульсного трансформатора. Кроме того рассматривается влияние паразитных индуктивностей на напряжение на нагрузочной емкости объекта контроля, а также на емкости накопительного конденсатора. Согласно результатам проведенного моделирования паразитная индуктивность объекта контроля оказывает более значительное влияние на форму напряжения на нагрузочной емкости, чем паразитная индуктивность накопительного конденсатора.

**Ключевые слова:** характеристический полином, взаимная индуктивность, преобразование Лапласа.

***I. KOSTIUKOV***

## THE INFLUENCE OF STRAY INDUCTANCE OF STORAGE CAPACITOR AND LOAD CAPACITANCE ON TRANSIENTS IN HIGH VOLTAGE FACILITIES WITH PULSE TRANSFORMERS

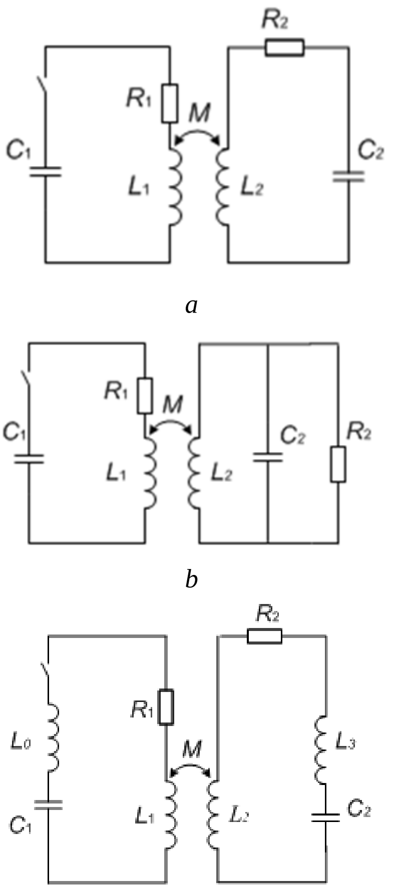
High voltage facilities based on gaining of voltage pulses by means of applying various types of pulse transformers are widely used in various engineering applications. Such high voltage facilities can be used for the elaboration of particle accelerators and in practice of forming pulses of high voltage for the purposes of electrical equipment testing. Technical characteristics of such high voltage facilities are significantly influenced by the technical characteristics of applied pulse transformer and, therefore, by the parameters of applied equivalent scheme of pulse transformer. Requirements to the obtained time dependencies of formed voltage pulses that arise in practice of testing high voltage insulation by applying pulses of high voltage demand to take into consideration the influence of parasitic parameters of equivalent scheme. This paper presents the results of consideration of the influence of parasitic inductances of storage capacitor and capacitance of tested object on currents in primary and secondary windings of pulse transformer. The influence of stray inductance on time dependence of voltage on load capacitance of tested object of control, as well as on time dependence of voltage on capacitance of storage capacitor is also considered. According to the results of carried out simulations, the inevitable parasitic inductance of tested object has more significant effect on time dependence of voltage on load capacitance than the parasitic inductance of storage capacitor.

**Ключові слова:** characteristic polynomial, mutual inductance, Laplace transform.

**Introduction.** High voltage pulse transformers are widely used in practice of forming pulses of high voltage in various engineering applications. Generators of high voltage with pulse transformers, for example, can be used for the elaboration of particle accelerators [1] and in practice of forming pulses of high voltage for the testing of electrical equipment [2]. For different purposes such transformers can be elaborated either with magnetic core, made of material with low value of power losses, or without any magnetic core. In case of properly chosen parameters of equivalent schemes, that fit specific ratios between the values of load capacitance, storage capacitance and values of inductances of primary and secondary windings, such high voltage facilities correspond to the scheme of Tesla transformer and can operate in a resonant mode. In some cases, especially which correspond to the operation of pulse transformer in a resonant mode, the analysis of transients in primary and secondary circuits often is carried out without taking into consideration energy dissipation in windings of pulse transformer [3-4]. Nevertheless, certain requirements to the obtained time dependencies of formed pulses on load capacitance, that arise in practice of testing high voltage insulation by applying pulses of high voltage, require more specified model, which allows to take into consideration the influence of power losses in primary and secondary windings on the value of voltage formed on load capacitance. Another factor that can cause impact on voltage pulses formed on capacitive load of tested object is determined by inevitable stray inductance of storage capacitance that leads to additional voltage drop in primary winding of pulse transformer. For high voltage capacitors that have been designed for operating in high voltage facilities for forming short pulses of voltage, the value of stray inductance usually is considered as a standardized parameter. For example, for such high voltage pulse capacitors as ИМ-40-0.9-У3 and ИМ-70-0.9-У3, according to the listed in [5] parameters, the value of stray inductance is equal to $300 \cdot 10^{-9}$ Hn. In general case parasitic inductance can contain components caused by the inductances of capacitor plates, internal connecting busses and outputs of high voltage capacitor [6]. Stray inductance of load capacitance, similarly to the stray inductance of storage capacitor, can cause influence on time dependence of voltage on load capacitance of tested object. This paper was motivated by the necessity of taking into consideration and clarifying of mentioned influence.

**The objective of this paper** is the modeling of the influence of parasitic inductance of storage capacitor and load capacitance on time dependencies of voltage drop on load capacitance.

**Lumped equivalent schemes of pulse transformer.** The analysis can be made on the basis of the following simplified equivalent schemes of pulse transformer:




*c*

Fig. 1. Equivalent schemes for the analysis of transients in primary and secondary windings, caused by the discharge of storage capacitor: a) with series connection of power loss resistance [3], b) with parallel connection of power loss resistance [7], c) with series connection of power loss resistance and stray inductance of storage capacitor and load capacitance.

On the equivalent schemes presented on Fig. 1 $R_1$ and $R_2$ denote power loss resistances that have been caused by power losses in primary and secondary windings, $L_1$ and $L_2$ denote the values of inductances of primary and secondary windings, $C_1$ and $C_2$ respectively denote the values of storage capacitor and load capacitance of tested capacitive object, $M$ is the value of mutual induction between the primary and secondary windings, $L_0$ denotes the value of stray inductance of storage capacitor, $L_3$ is the value of stray inductance of load capacitance. Taking into consideration the presence of skin-effect in wires of primary and secondary windings such parameters of described equivalent schemes as $L_1$, $L_2$, $R_1$, $R_2$ can exhibit some dependence on spectral properties of discharge current. The modeling of the influence of stray capacitance of tested object and storage capacitor was carried out under the assumption of negligible possible influence of non-uniform distribution of currents in wires of primary and secondary windings on the parameters of described equivalent schemes. The analysis also is to be hold for the case of air pulse transformer, therefore the value of mutual inductance $M$ can considered to be independent on magnetic properties of core material.

It should be noted that the equivalent schemes on Fig. 1 have been presented with some simplifications, as they do not take into consideration the presence of stray capacitance between primary and secondary windings and stray capacitances of primary and secondary windings with respect to the ground.

As it can be noticed, the main distinction between the equivalent schemes described on Fig.1 *a* Fig.1 *b* is caused by the fact that power losses in secondary circuit can be taken into consideration either by series connection or by parallel connection of power loss resistance $R_2$. Current analysis will be made for the case of series connection of $R_2$ and $C_2$. Comprehensive analysis of transients in this equivalent scheme has also been carried out in [8]. However, mentioned analysis doesn't contain the consideration of issues caused by stray inductance of storage capacitor.

**The influence of stray inductance of storage capacitor on currents in primary and secondary winding.** For lumped equivalent scheme on Fig. 1 *a* in the case of negligible stray inductance of tested object ($L_3 = 0$) the system of equations that determine the values of voltage drop on every element of equivalent scheme can be written in the following form:

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt - M\frac{di_2}{dt} + L_0\frac{di_1}{dt} = U_0; \qquad (1)$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2 dt - M\frac{di_1}{dt} = 0. \qquad (2)$$

where $U_0$ is the value of voltage on storage capacitance. Laplace transform of terms in (1) and (2) can be written in the following form:

$$pL_1 i_1 + R_1 i_1 + \frac{i_1}{pC_1} - pMi_2 + pL_0 i_1 = \frac{U_0}{p}; \qquad (3)$$

$$pL_2 i_2 + R_2 i_2 + \frac{i_2}{pC_2} - pMi_1 = 0. \qquad (4)$$

Therefore, the expression for Laplace transform for currents in primary and secondary winding of pulse transformer can be written in the following form:

$$i_1(p) = \frac{U_0 C_1 (L_2 C_2 p^2 + R_2 C_2 p + 1)}{a_1 p^4 + b_1 p^3 + c_1 p^2 + d_1 p + 1}; \qquad (5)$$

$$i_2(p) = \frac{p^2 M U_0 C_1 C_2}{a_1 p^4 + b_1 p^3 + c_1 p^2 + d_1 p + 1}; \qquad (6)$$

where all constants $a_1$ $b_1$ $c_1$ $d_1$ can be calculated by using the following expressions:

$$a_1 = C_1C_2((L_1 + L_0)L_2 - M^2); \quad (7)$$

$$b_1 = C_1C_2(L_2R_1 + R_2(L_1 + L_0)); \quad (8)$$

$$c_1 = (R_2C_2R_1 + L_1 + L_0)C_1 + L_2C_2; \quad (9)$$

$$d_1 = R_1C_1 + R_2C_2. \quad (10)$$

It can be noticed, that the influence of stray inductance of storage capacitor leads to increasing of coefficients $a_1$, $b_1$, $c_1$, whereas coefficient $d_1$ is not subjected to the influence of stray inductance of storage capacitor.

Conventional scheme of deriving time dependencies for currents in primary and secondary windings implies the necessity of obtaining the roots of the following polynomial:

$$a_1p^4 + b_1p^3 + c_1p^2 + d_1p + 1 = 0. \quad (11)$$

In practice of analysis transients in electrical circuits calculated roots of such algebraic equations usually can be obtained in a complex form with negative real parts, as a consequence of currents attenuation. For the case of highly damped oscillations imaginary part of roots can be negligible in comparison with real part.

In general form time dependence of voltage on storage capacitance can be determined by using the following expression:

$$u_{C_1}(t) = \frac{1}{C_1}\int_0^t i_1(t)dt = \frac{1}{C_1}\sum_{n=1}^{4}\frac{N(p_n)}{M'(p_n)}\cdot\left(\frac{e^{p_nt}}{p_n} - \frac{1}{p_n}\right); \quad (12)$$

where all $p_n$ denote the roots of characteristic polynomial (11), $N(p_n)$ denotes the values of nominator in (5) in points that correspond to the roots of characteristic polynomial (11), $M'(p_n)$ denotes the values of derivative from denominator in (5) in points that correspond to the roots of characteristic polynomial (11).

Similarly to the previous expression, the time dependence of voltage on the load capacitance of tested object can be determined by using the following expression:

$$u_{C_2}(t) = \frac{1}{C_2}\int_0^t i_2(t)dt = \frac{1}{C_2}\sum_{n=1}^{4}\frac{B(p_n)}{Q'(p_n)}\cdot\left(\frac{e^{p_nt}}{p_n} - \frac{1}{p_n}\right) \quad (13)$$

where $B(p_n)$ denotes the values of nominator in (6) in points that correspond to the roots of characteristic polynomial (11), $Q'(p_n)$ denotes the values of derivative from denominator in (6) in points that correspond to the roots of characteristic polynomial (11).

**The influence of stray inductance of load capacitance on currents in primary and secondary winding.** The case of substantial stray inductance of load capacitance corresponds to the equivalent scheme presented on Fig. 1 *c*. For the case of significant stray inductance of tested object, coefficients in polynomial (11) should be, obviously, written in the following form:

$$a_2 = C_1C_2((L_2 + L_3)L_0 + (L_2 + L_3)L_1 - M^2) \quad (14)$$

$$b_2 = C_1C_2(L_0R_2 + R_2L_1 + R_1(L_2 + L_3)) \quad (15)$$

$$c_2 = (R_2C_2R_1 + L_1 + L_0)C_1 + (L_2 + L_3)C_2 \quad (16)$$

The distinction between (7)-(9) and (14)-(16) will be determined only by the presence of additional term, caused by parasitic inductance of tested object. Therefore, all calculations can easily be carried out by applying expressions (12) and (13), but by using the roots of characteristic polynomial with coefficients $a_2$, $b_2$, $c_2$ instead of $a_1$ $b_1$ $c_1$. It can be seen that for the case of negligible stray inductance of load capacitance expressions (14-16) can easily be reduced to the previously considered case of constants $a_1$ $b_1$ $c_1$.

Fig. 2 and Fig. 3 present the results of mathematical modeling of time dependencies of voltage on load capacitance. Fig. 4 and Fig. 5 present the influence of stray inductance on the roots of algebraic equation (11). All calculations have been carried out for the value of voltage on storage capacitance equal to 10 kV. The value of mutual inductance between the primary and secondary winding was set equal to $1.133\cdot10^{-4}$ Hn. The values of inductance of primary and secondary windings have been chosen equal to $182\cdot10^{-6}$ Hn and $126\cdot10^{-6}$ Hn. The values of storage capacitor and load capacitance have been respectively chosen equal to 3 μF and 2 nF.

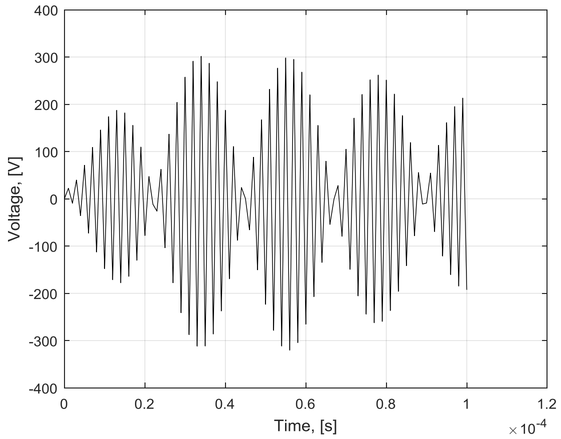



Fig. 2. Difference between voltages on load capacitance, calculated for the case of $L_0 = 0$, $L_3 = 0$ and for the case of $L_0 = 300\cdot10^{-9}$ Hn and $L_3 = 0$.

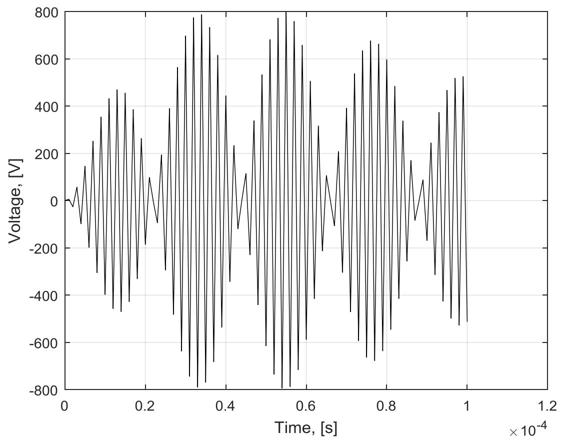



Fig. 3. Difference between voltages on load capacitance calculated for the case of $L_0 = 0$, $L_3 = 0$ and for the case of $L_0 = 0$, $L_3 = 300$ nHn.

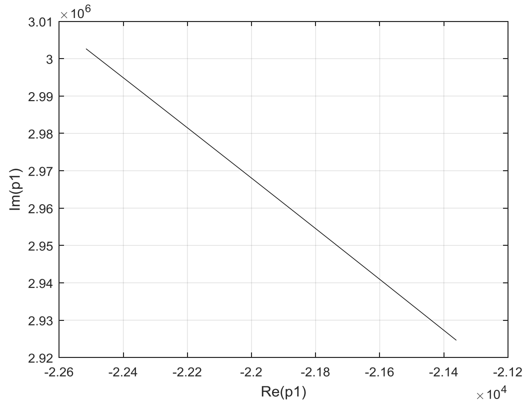



Fig. 4. Real and imaginary components of characteristic polynomial roots for the case of $L_0$ = 0 and $L_3$ considered to be variable in a range from 0 to $300{\cdot}10^{-8}$ Hn.

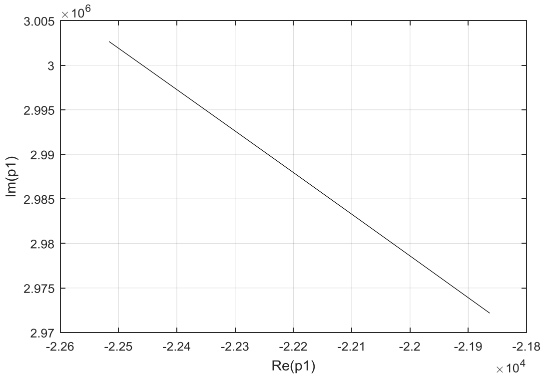



Fig.5. Real and imaginary components of characteristic polynomial roots for the case of $L_3$ = 0 and $L_0$ considered to be variable in a range from 0 to $300{\cdot}10^{-8}$ Hn.

As it can be concluded from the results of calculations on Fig. 2 and Fig. 3, the presence of stray inductance of tested object causes more significant impact on time dependence of voltage drop on tested capacitive object in comparison with stray inductance of storage capacitor. While for the case of $L_0$ = 300 nHn and $L_3$ = 0 the highest difference in voltages riches 0.3 kV, for the case of $L_0$ = 0 and $L_3$ = 300 nHn the highest difference in voltages riches 0.8 kV. As it can also be concluded from the results of modelling presented on Fig. 4 and Fig. 5, stray inductances of storage capacitor and load capacitance cause some distinction in obtained roots of characteristic polynomial.

**Conclusions.** Carried out analysis illustrates the influence of stray inductance of load capacitance and storage capacitor on time dependence of voltage on load capacitance of tested object. Both sources of parasitic inductance can be taken into consideration by increasing the inductances of primary and secondary windings in corresponding terms of characteristic polynomial. According to the obtained results, stray inductance of tested object causes more significant influence on time dependence of voltage on load capacitance in comparison with stray inductance of storage capacitor.

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